

Let  $f(x) = \cos^{-1} \frac{2}{x}$ .

SCORE: \_\_\_\_\_ / 20 PTS

[a] If  $x$  changes from 4 to 3.8, find  $dy$ .

$$f'(x) = \left[ -\frac{1}{\sqrt{1-\left(\frac{2}{x}\right)^2}} \right] \cdot \left[ -\frac{2}{x^2} \right] \textcircled{4}$$

$$\begin{aligned} f'(4) &= -\frac{1}{\sqrt{1-\frac{1}{4}}} \cdot -\frac{2}{16} \\ &= \frac{2}{\sqrt{3}} \cdot \frac{1}{8} = \left[ \frac{1}{4\sqrt{3}} \right] \textcircled{2} \end{aligned}$$

$$dy = \frac{1}{4\sqrt{3}} (3.8-4)$$

$$= \frac{1}{4\sqrt{3}} \cdot \left[ -\frac{1}{5} \right] \textcircled{3}$$

$$= \left[ -\frac{1}{20\sqrt{3}} \right] \textcircled{2}$$

[b] Approximate  $f(3.8)$  using your answer to part [a].

$$f(3.8) \approx f(4) + dy = \cos^{-1} \frac{2}{4} - \frac{1}{20\sqrt{3}} = \left[ \frac{\pi}{3} \right] \textcircled{2\frac{1}{2}} - \left[ \frac{1}{20\sqrt{3}} \right] \textcircled{2\frac{1}{2}}$$

The position of an object at time  $t$  minutes is given by the function  $s(t) = \frac{t^3 - 9t + 15}{10\sqrt[3]{t}}$  yards for  $t \geq 0.5$ .

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Find the acceleration of the object at time  $t = 1$  minute.

$$s(t) = \frac{1}{10}t^{\frac{8}{3}} - \frac{9}{10}t^{\frac{2}{3}} + \frac{3}{2}t^{-\frac{1}{3}}$$

$$s'(t) = \frac{4}{15}t^{\frac{5}{3}} - \frac{3}{5}t^{-\frac{1}{3}} - \frac{1}{2}t^{-\frac{4}{3}} \quad (5)$$

$$s''(t) = \frac{4}{9}t^{\frac{2}{3}} + \frac{1}{5}t^{-\frac{4}{3}} + \frac{2}{3}t^{-\frac{7}{3}} \quad (5)$$

$$s''(1) = \frac{4}{9} + \frac{1}{5} + \frac{2}{3} = \frac{20 + 9 + 30}{45} = \frac{59}{45} \text{ YARDS/MINUTE}^2 \quad (2)$$

(3)

The following table gives values and derivatives of a function at various inputs.

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$x$	-3	-2	-1	0	1	2	3	4
$f(x)$	-2	0	2	4	-3	-1	1	3
$f'(x)$	4	-1	-3	2	-4	3	-2	1

If  $m(x) = 2^{f(x)}$ , find the equation of the tangent line to  $y = m(x)$  at  $x = -3$ .

$$m(-3) = 2^{f(-3)} = 2^{-2} = \frac{1}{4} \quad (3)$$

$$m'(x) = 2^{f(x)} (\ln 2) f'(x) \quad (6)$$

$$m'(-3) = 2^{f(-3)} (\ln 2) f'(-3) = 2^{-2} (\ln 2) 4 = \ln 2 \quad (3)$$

$$y - \frac{1}{4} = (\ln 2)(x + 3) \quad (3)$$

If  $\cos y^2 - \sin^2 x = x^3 e^{2y}$ , find  $\frac{dy}{dx}$ .

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$$\boxed{(-\sin y^2) 2y \frac{dy}{dx}} - \boxed{2 \sin x \cos x} = \boxed{3x^2 e^{2y}} + \boxed{x^3 (2e^{2y} \frac{dy}{dx})}$$

$$-2 \sin x \cos x - 3x^2 e^{2y} = (2x^3 e^{2y} + 2y \sin y^2) \frac{dy}{dx}$$

$$\boxed{-\frac{2 \sin x \cos x + 3x^2 e^{2y}}{2x^3 e^{2y} + 2y \sin y^2}} = \frac{dy}{dx}$$

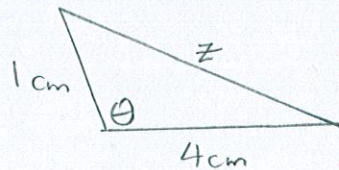
(6)

Two sides of a triangle have lengths 1 cm and 4 cm. At the moment when the angle between those two sides is  $120^\circ$ , that angle is shrinking by  $10^\circ$  per minute. How quickly is length of the third side changing at that moment? SCORE: \_\_\_\_\_ / 25 PTS

You must state whether the third side is getting longer or shorter.

You must state/show clearly what each variable you use represents.

You must show the units during the intermediate steps of your work, and you must state the units for the final answer.



$$\textcircled{5} \quad \begin{aligned} z^2 &= (1 \text{ cm})^2 + (4 \text{ cm})^2 - 2(1 \text{ cm})(4 \text{ cm}) \cos \theta \\ &= (17 - 8 \cos \theta) \text{ cm}^2 \end{aligned} \quad \textcircled{1}$$

$$\textcircled{5} \quad \frac{d}{dt} z^2 = \frac{dz}{dt} \cdot 2z = 8(\sin \theta) \frac{d\theta}{dt} \text{ cm}^2$$

$$\frac{d\theta}{dt} \Big|_{\theta = \frac{2\pi}{3}} = -\frac{\pi}{18} / \text{MINUTE}$$

$$\text{WANT } \frac{dz}{dt} \Big|_{\theta = \frac{2\pi}{3}}$$

$$\textcircled{2} \quad \theta = \frac{2\pi}{3}$$

$$z^2 = (17 - 8 \cos \frac{2\pi}{3}) \text{ cm}^2$$

$$= 21 \text{ cm}^2$$

$$z = \sqrt{21} \text{ cm}$$

$$\sqrt{21} \text{ cm} \frac{dz}{dt} = 4(\sin \frac{2\pi}{3}) \left( -\frac{\pi}{18} / \text{MINUTE} \right) \text{ cm}^2 \quad \textcircled{2}$$

$$\textcircled{3} \quad \frac{dz}{dt} = -4 \left( \frac{\sqrt{3}}{2} \right) \left( \frac{\pi}{18} \right) \left( \frac{1}{\sqrt{21}} \right) \frac{\text{cm}}{\text{MINUTE}}$$

$$= \frac{-\pi}{9\sqrt{7}} \frac{\text{cm}}{\text{MINUTE}} \quad \textcircled{3}$$

THE THIRD SIDE IS GETTING SHORTER  $\textcircled{2}$

BY  $\frac{\pi}{9\sqrt{7}}$  cm PER MINUTE  $\textcircled{1}$

$\textcircled{1}$

Find  $\frac{d}{dx}(\csc x)^{\tan x}$ .

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$$y = (\csc x)^{\tan x}$$

$$\textcircled{3} \ln y = \tan x \ln \csc x$$

$$\textcircled{1} \frac{1}{y} \frac{dy}{dx} = \textcircled{2} \sec^2 x \ln \csc x + \textcircled{4} \tan x \frac{1}{\csc x} (-\csc x \cot x) = \sec^2 x \ln \csc x - 1$$

$$\textcircled{2} \frac{dy}{dx} = y (\textcircled{2} \sec^2 x \ln \csc x - 1) = \textcircled{2} (\csc x)^{\tan x} (\textcircled{2} \sec^2 x \ln \csc x - 1)$$

Prove the quotient rule using the definition of the derivative function. Show all steps.

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$$\left(\frac{f(x)}{g(x)}\right)' = \lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \quad (2)$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x+h)g(x)} \quad (2)$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) + f(x)g(x) - f(x)g(x) - f(x)g(x+h)}{hg(x+h)g(x)} \quad (3)$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \cdot \lim_{h \rightarrow 0} g(x) + \lim_{h \rightarrow 0} f(x) \cdot \lim_{h \rightarrow 0} \frac{g(x) - g(x+h)}{h} \quad (2\frac{1}{2}) \quad (2\frac{1}{2})$$
$$\lim_{h \rightarrow 0} g(x+h)g(x) \quad (2)$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \quad (1)$$

If  $f(x) = \tan^{-1} x^3$ , find  $f''(-1)$ .

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$$f'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2 = \frac{3x^2}{1+x^6}$$

$$f''(x) = \frac{6x(1+x^6) - 3x^2(6x^5)}{(1+x^6)^2}$$

$$f''(-1) = \frac{-6(2) - 3(-6)}{2^2} = \frac{3}{2}$$